Hold on to your homework today!

(We will turn in at the end of class.)

Review of vector stuff

- What is the dot product useful for? (result = a number)
  - $\|v\| = |v| =$ length of $v = \sqrt{v \cdot v}$.
  - Can find the angle between two vectors using

$v \cdot w = \|v\| \, \|w\| \cos\theta$.

  - Two vectors v, w are $\perp$ $\iff$ $v \cdot w = 0$.

- What is the cross product $v \times w$ useful for? (result → a vector)

- $v \times w$ is $\perp$ to both v & w.
- $|v \times w|$ = area of parallelogram spanned by v & w

- $|v \times w| = |v| \cdot |w| \cdot \sin\theta$
- only useful in 3-d.

[in future: something called "differential forms" will allow us to do something in higher dimensions]

- $v \times w = \vec{0} \Leftrightarrow v, w$ are in same or opposite direction.

- What is the determinant of a matrix useful for?

- $\det(A) = \pm$ area of parallelogram spanned by column vectors

$= \begin{vmatrix} v_1 & w_1 \\ v_2 & w_2 \end{vmatrix}$

2×2 case

shocking

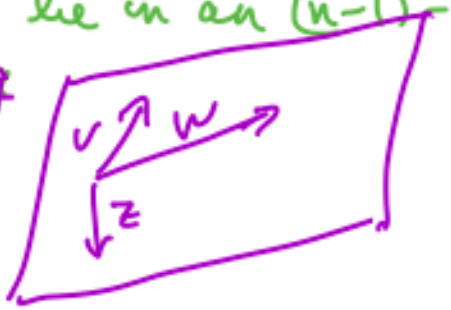
$=$ also $\pm$ area of parallelogram spanned by row vectors.

- $\det(A) = \pm$ volume of the parallelepiped spanned by the column vectors.

nxn case $n \geq 3$.

- $\det(A) = 0$ if the columns lie in an $(n-1)$-dimensional space.

eg



Example Consider the plane

$$z = 4x - 7y + 11.$$

Find some vectors parallel to this plane.

- What can we get from the equation?

A Normal vector to the plane:

eqn: $0 = 4x - 7y - z + 11$

$N = (4, -7, -1)$.

- What must be true about vectors parallel to the plane?

must be $\perp$ to N.

- How do we find vectors $\perp$ to our N?

→ Just find vectors that dot with N and get 0.

$(v_1, v_2, v_3) \cdot (4, -7, -1) = 0$

eg $(2,1,1)$. ✓

$(7,4,0)$. ✓

$(0,1,-7)$. ✓

Find the angle between the planes

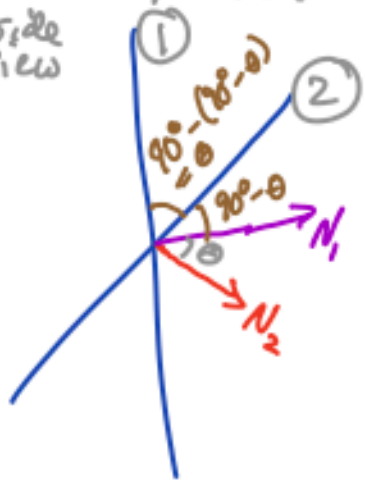
$x + 3y - 7z = 12$ ①

and $2x + y + z = 4$ ②

- What do I know about these planes?

① $N_1 = (1, 3, -7)$

② $N_2 = (2, 1, 1)$



- Angle between ① & ② = Angle between N_1 & $N_2 = \theta$

$N_1 \cdot N_2 = |N_1|\,|N_2| \cos\theta$

$2 + 3 - 7 = \sqrt{1+9+49}\sqrt{4+1+1}\cos\theta$

$\sqrt{59}$ $\sqrt{6}$

$$\frac{-2}{\sqrt{6}\sqrt{59}} = \cos\theta \Rightarrow \theta = \arccos\left(\frac{-2}{\sqrt{6}\sqrt{59}}\right)$$

Note: Angle is $> 90°$ $\rightsquigarrow$ so smaller angle between the planes $= \pi - \theta$

is $180^0 - \arccos\left(\frac{-2}{\sqrt{6}\sqrt{54}}\right) = \arccos\left(\frac{2}{\sqrt{6}\sqrt{54}}\right)$.